PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO
INSTITUTO DE MATEMÁTICAS

PAUTA PRUEBA Nº1 MAT-213

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I)

$$F_{41}(-2)\cdot F_4(2)\cdot F_{23}\cdot X\cdot B=C$$
$$\Rightarrow\quad X=F_{23}^{-1}\cdot F_4^{-1}(2)\cdot F_{41}^{-1}(-2)\cdot C\cdot B^{-1}$$
$$\Rightarrow\quad X=F_{23}\cdot F_4\left(\frac{1}{2}\right)\cdot F_{41}(2)\cdot C\cdot B^{-1}$$

Como $|B|\overset{F_3}{=}\begin{vmatrix}1&0\\-1&2\end{vmatrix}=2\neq 0$, se tiene que B es invertible.

$$\left[B\,\middle|\,I_3\right]=\left[\begin{array}{ccc|ccc}2&1&0&1&0&0\\0&-1&2&0&1&0\\1&0&0&0&0&1\end{array}\right]\xrightarrow{F}\left[\begin{array}{ccc|ccc}1&0&0&0&0&1\\0&1&0&1&0&-2\\0&0&1&1/2&1/2&-1\end{array}\right]=\left[I_3\,\middle|\,B^{-1}\right]$$

$$\therefore\ CB^{-1}=\begin{pmatrix}4&0&3\\0&1&5\\-1&0&0\\-2&-2&0\end{pmatrix}\begin{pmatrix}0&0&1\\1&0&-2\\1/2&1/2&-1\end{pmatrix}=\begin{pmatrix}3/2&3/2&1\\7/2&5/2&-7\\0&0&-1\\-2&0&2\end{pmatrix}$$

$$CB^{-1}\xrightarrow{F_{41}(2)}\begin{pmatrix}3/2&3/2&1\\7/2&5/2&-7\\0&0&-1\\1&3&4\end{pmatrix}\xrightarrow{F_4\left(\frac{1}{2}\right)}\begin{pmatrix}3/2&3/2&1\\7/2&5/2&-7\\0&0&-1\\1/2&3/2&2\end{pmatrix}\xrightarrow{F_{23}}\begin{pmatrix}3/2&3/2&1\\0&0&-1\\7/2&5/2&-7\\1/2&3/2&2\end{pmatrix}$$

$$\therefore\ X=\begin{pmatrix}3/2&3/2&1\\0&0&-1\\7/2&5/2&-7\\1/2&3/2&2\end{pmatrix}$$

II) $$|A|=\begin{vmatrix}0&a&b\\a&0&b\\b&a&0\end{vmatrix}=-a\begin{vmatrix}a&b\\a&0\end{vmatrix}+b\begin{vmatrix}a&b\\0&b\end{vmatrix}=a^2b+ab^2=ab(a+b)$$

A es invertible $\Leftrightarrow |A|\neq 0\Leftrightarrow ab(a+b)\neq 0\Leftrightarrow (a\neq 0)\wedge(b\neq 0)\wedge(a\neq -b)$

III)

$$\left|P^2\right|=\left|F_{12}(-2)\cdot F_3(2)\cdot F_1(-2)\cdot F_{13}\cdot Q\right|$$
$$\Rightarrow |P|^2=\left|F_{12}(-2)\right|\cdot\left|F_3(2)\right|\cdot\left|F_1(-2)\right|\cdot\left|F_{13}\right|\cdot|Q|$$
$$\Rightarrow |P|^2=1\cdot 2\cdot(-2)\cdot(-1)\cdot|Q|$$
$$\therefore\ |Q|=\frac{1}{4}\cdot|P|^2$$

pero $$|P|=\begin{vmatrix}1&1&2&3\\0&1&2&3\\0&0&-2&-3\\0&0&-3&-6\end{vmatrix}=\begin{vmatrix}1&2&3\\0&-2&-3\\0&-3&-6\end{vmatrix}=\begin{vmatrix}-2&-3\\-3&-6\end{vmatrix}=3$$

$$\therefore |Q|=\frac{9}{4}$$

IV)

$$\Delta = \begin{vmatrix} 1 & 0 & 2 & -1 \\ -1 & 2 & 2a & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & a \end{vmatrix} \xrightarrow{F} \begin{vmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & 2a & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -5 & a+2 \end{vmatrix} = \begin{vmatrix} 0 & 2a & 0 \\ 1 & 1 & 0 \\ 0 & -5 & a+2 \end{vmatrix} = -\left[2a(a+2)\right]$$

1) El sistema tiene única solución si $a \neq 0 \ \wedge \ a \neq -2$.

Para $a = 1$, se tiene: $\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ -1 & 2 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 \end{array}\right) \xrightarrow{F} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1/3 \end{array}\right)$

$$\therefore \quad x = \frac{2}{3}; y = z = 0; w = -\frac{1}{3} \qquad \therefore S = \left\{ \begin{pmatrix} 2/3 \\ 0 \\ 0 \\ -1/3 \end{pmatrix} \right\}$$

2) Si $a = -2$, se tiene: $\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ -1 & 2 & -4 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & -2 & 1 \end{array}\right) \xrightarrow{F} \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right)$, luego

el sistema no tiene solución, pues $Rg(E) < Rg(E|b)$.

3) Si $a = 0$, se tiene: $\left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ -1 & 2 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{F} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1/5 & 3/5 \\ 0 & 1 & 0 & 2/5 & -1/5 \\ 0 & 0 & 1 & -2/5 & 1/5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$,

luego el sistema tiene infinitas soluciones, pues $Rg(E) = Rg(E|b) = 2 < 4$. En este caso el conjunto solución es:

$$S = \left\{ \left(\frac{3}{5} + \frac{1}{5}t, -\frac{1}{5} - \frac{2}{5}t, \frac{1}{5} + \frac{2}{5}t, t \right)^T ; \quad t \in \mathbb{R} \right\}$$